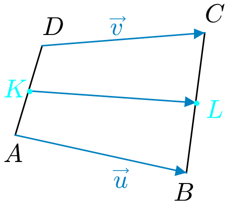
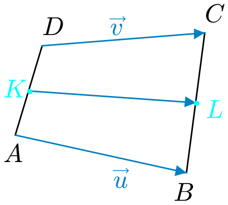
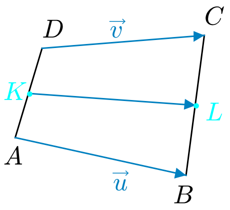
Příklad 7

Je dán konvexní čtyřúhelník $ABCD$, jehož strany AB a CD jsou shodné. Dokažte, že přímky AB a CD svírají stejný úhel s přímkou, která prochází středy stran AD a BC.



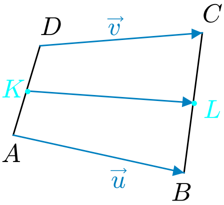


$$\overrightarrow{KL} = L - K = \frac{B+C}{2} - \frac{A+D}{2} = \frac{(B-A)+(C-D)}{2} = \frac{\vec{u}+\vec{v}}{2},$$



$$\overrightarrow{KL} = L - K = \frac{B+C}{2} - \frac{A+D}{2} = \frac{(B-A)+(C-D)}{2} = \frac{\vec{u}+\vec{v}}{2},$$

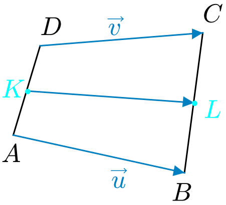
$$\cos\left(\vec{u}, \overrightarrow{KL}\right) = \frac{\vec{u}\cdot(\vec{u}+\vec{v})}{|\vec{u}|\cdot|\vec{u}+\vec{v}|} = \frac{|\vec{u}|^2 + \vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{u}+\vec{v}|},$$



$$\overrightarrow{KL} = L - K = \frac{B+C}{2} - \frac{A+D}{2} = \frac{(B-A)+(C-D)}{2} = \frac{\vec{u}+\vec{v}}{2},$$

$$\cos\left(\vec{u}, \overrightarrow{KL}\right) = \frac{\vec{u}\cdot(\vec{u}+\vec{v})}{|\vec{u}|\cdot|\vec{u}+\vec{v}|} = \frac{|\vec{u}|^2+\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{u}+\vec{v}|},$$

$$\cos\left(\vec{v}, \overrightarrow{KL}\right) = \frac{\vec{v}\cdot(\vec{u}+\vec{v})}{|\vec{v}|\cdot|\vec{u}+\vec{v}|} = \frac{|\vec{v}|^2+\vec{u}\cdot\vec{v}}{|\vec{v}|\cdot|\vec{u}+\vec{v}|.}$$



$$\overrightarrow{KL} = L - K = \frac{B+C}{2} - \frac{A+D}{2} = \frac{(B-A)+(C-D)}{2} = \frac{\vec{u}+\vec{v}}{2},$$

$$\cos\left(\vec{u}, \overrightarrow{KL}\right) = \frac{\vec{u}\cdot(\vec{u}+\vec{v})}{|\vec{u}|\cdot|\vec{u}+\vec{v}|} = \frac{|\vec{u}|^2+\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{u}+\vec{v}|},$$

$$\cos\left(\vec{v}, \overrightarrow{KL}\right) = \frac{\vec{v}\cdot(\vec{u}+\vec{v})}{|\vec{v}|\cdot|\vec{u}+\vec{v}|} = \frac{|\vec{v}|^2+\vec{u}\cdot\vec{v}}{|\vec{v}|\cdot|\vec{u}+\vec{v}|.}$$

Díky předpokladu $|\vec{u}| = |\vec{v}|$ se oba kosiny rovnají. $\square$