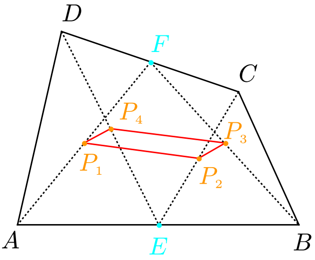
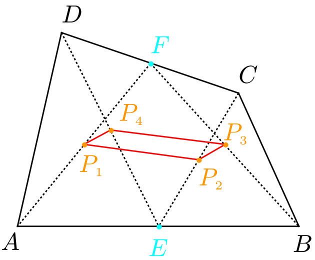
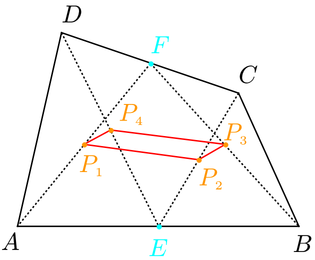
Příklad 1

Ve čtyřúhelníku $ABCD$, jehož strany AB a CD nejsou rovnoběžné, označíme E střed strany AB a F střed strany CD. Dokažte, že středy úseček AF, CE, BF, DE jsou vrcholy rovnoběžníku.





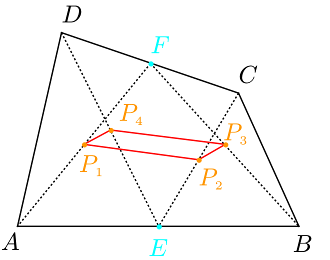
$$E = \frac{1}{2}(A + B), \quad F = \frac{1}{2}(C + D)$$



$$E = \frac{1}{2}(A+B), \quad F = \frac{1}{2}(C+D)$$

$$P_1 = \frac{1}{2}(A+F) = \frac{1}{4}(2A+C+D), \qquad P_2 = \frac{1}{4}(2C+A+B),$$

$$P_3 = \frac{1}{4}(2B+C+D), \qquad P_4 = \frac{1}{4}(2D+A+B)$$

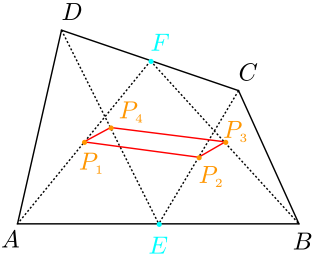


$$E = \frac{1}{2}(A+B), \quad F = \frac{1}{2}(C+D)$$

$$P_1 = \frac{1}{2}(A+F) = \frac{1}{4}(2A+C+D), \qquad P_2 = \frac{1}{4}(2C+A+B),$$

$$P_3 = \frac{1}{4}(2B+C+D), \qquad P_4 = \frac{1}{4}(2D+A+B)$$

$$P_2 - P_1 = \frac{1}{4}(-A+B+C-D) = P_3 - P_4$$

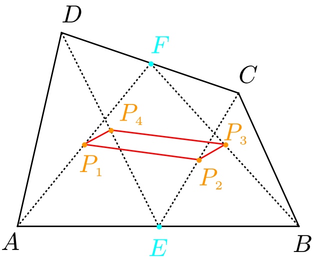


$$E = \frac{1}{2}(A + B), \quad F = \frac{1}{2}(C + D)$$

$$P_1 = \frac{1}{2}(A + F) = \frac{1}{4}(2A + C + D), \qquad P_2 = \frac{1}{4}(2C + A + B),$$

$$P_3 = \frac{1}{4}(2B + C + D), \qquad P_4 = \frac{1}{4}(2D + A + B)$$

$$P_2 - P_1 = \frac{1}{4}(-A + B + C - D) = P_3 - P_4 = \frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{CD} \neq \vec{o}$$



$$\overrightarrow{P_1P_2} = \overrightarrow{P_4P_3} = +\frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{CD},$$

$$\overrightarrow{P_4P_1} = \overrightarrow{P_3P_2} = -\frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{CD} \quad \square$$