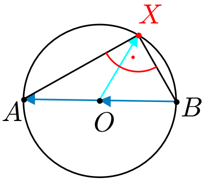
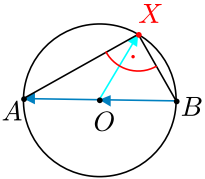
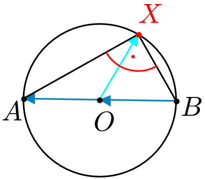
Příklad 6

Dokažte Thaletovu větu: *Je-li bod O střed úsečky AB o délce $2r$, pak pro každý bod X různý od bodů A, B je úhel AXB pravý, právě když platí $|OX| = r$.*



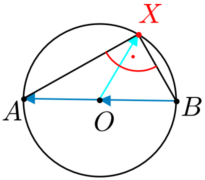


$$\overrightarrow{OA} = \overrightarrow{BO} \quad \text{a} \quad |\overrightarrow{OA}| = |\overrightarrow{BO}| = r$$



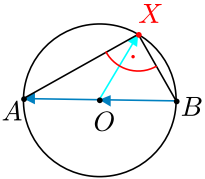
$\overrightarrow{OA} = \overrightarrow{BO}$ a $|\overrightarrow{OA}| = |\overrightarrow{BO}| = r$

$$|OX| = r \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = r^2 \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = |\overrightarrow{OA}|^2 \quad \Leftrightarrow$$



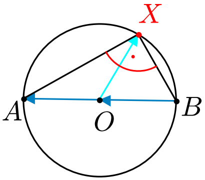
$$\overrightarrow{OA} = \overrightarrow{BO} \quad \text{a} \quad |\overrightarrow{OA}| = |\overrightarrow{BO}| = r$$

$$|OX| = r \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = r^2 \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = |\overrightarrow{OA}|^2 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \overrightarrow{OX} \cdot \overrightarrow{OX} - \overrightarrow{OA} \cdot \overrightarrow{OA} = 0 \quad \Leftrightarrow$$



$\overrightarrow{OA} = \overrightarrow{BO}$ a $|\overrightarrow{OA}| = |\overrightarrow{BO}| = r$

$$|OX| = r \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = r^2 \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = |\overrightarrow{OA}|^2 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \overrightarrow{OX} \cdot \overrightarrow{OX} - \overrightarrow{OA} \cdot \overrightarrow{OA} = 0 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \left(\overrightarrow{OX} - \overrightarrow{OA}\right) \cdot \left(\overrightarrow{OX} + \overrightarrow{OA}\right) = 0 \quad \Leftrightarrow$$



$$\overrightarrow{OA} = \overrightarrow{BO} \quad \text{a} \quad |\overrightarrow{OA}| = |\overrightarrow{BO}| = r$$

$$|OX| = r \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = r^2 \quad \Leftrightarrow \quad |\overrightarrow{OX}|^2 = |\overrightarrow{OA}|^2 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \overrightarrow{OX} \cdot \overrightarrow{OX} - \overrightarrow{OA} \cdot \overrightarrow{OA} = 0 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \left(\overrightarrow{OX} - \overrightarrow{OA}\right) \cdot \left(\overrightarrow{OX} + \overrightarrow{OA}\right) = 0 \quad \Leftrightarrow$$
$$\Leftrightarrow \quad \left(\overrightarrow{AO} + \overrightarrow{OX}\right) \cdot \left(\overrightarrow{BO} + \overrightarrow{OX}\right) = 0 \quad \Leftrightarrow \quad \overrightarrow{AX} \cdot \overrightarrow{BX} = 0 \quad \square$$