

Středověké astronomické tabulky



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Obsah

- Ekvace Slunce
- Ekvace Měsíce
- Časová rovnice
- Trepidace

The image shows a page from an astronomical manuscript. At the top, the title "Ejso lune" is written in red ink. Below the title, there is a table of numbers arranged in columns. The first column contains numbers from 0 to 120 in increments of 4. The subsequent columns contain various numerical values, some of which are written in red ink. The table is organized into several columns, with a vertical line separating the first column from the rest. The numbers are arranged in a grid-like pattern, typical of astronomical tables from the 16th or 17th century.

Paulerinus - Liber viginti artium, BJ 257

JHA 38 (2007), 305; DVT 41 (2008), 65

Astronomie = pátá věda,
fol. 131ra-142vb, tabulky 143r-152v

f. 131va: typy astronomie: přirozená, získaná, teoretická, ruční, **tabulková**, strojová, komputistická, kostelní

“**Tabulková astronomie** je založena na Alfonsinských nebo Lineriových tabulkách, na Anglických tabulkách nebo mých, které jsou velmi krátké a zabývají se během planet podle obecných či fyzických znamení”

f. 135 vb: “Abych však příliš nezlehčoval tuto přeslavnou vědu nebo ji dokonce nezlehčil natolik, že by ji mohli znevažovat i hlupáci: z žádné vědy jsem nebyl tak unaven a z žádné mne nebolela hlava a oči tolik, jako z této”



Ekvace Slunce

$$\epsilon = \arcsin \left(\frac{e \sin a}{\sqrt{1+2e \cos a + e^2}} \right)$$

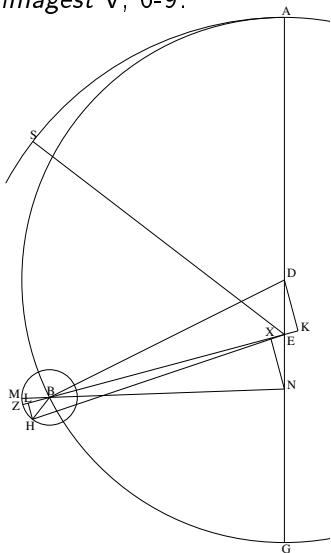
Tabula equationū Solis Prima

Sine signi	Subtrahere a			Subtrahere a			Subtrahere a			Sine signi			
	Equatio Solis	dr̄a eq̄at̄	dr̄a eq̄at̄	Equatio Solis	dr̄a eq̄at̄	dr̄a eq̄at̄	Equatio Solis	dr̄a eq̄at̄	dr̄a eq̄at̄				
	Gr̄	Min	Sec	Gr̄	Min	Sec	Gr̄	Min	Sec	Gr̄	Min	Sec	
1	0	2	10	2	9	10	1	2	26	1	41	41	29
2	0	2	19	2	8	19	1	6	31	1	42	46	28
3	0	6	24	2	9	24	1	8	28	1	42	0	24
4	0	8	36	2	8	36	1	10	19	1	40	6	26
5	0	10	44	2	9	44	1	12	9	1	46	9	24
6	0	12	47	2	9	47	1	13	46	1	44	11	22

X B 3 fol. 95v

Ekvace Měsíce

Almagest V, 6-9:



$$R \equiv DB, e \equiv DE, \eta \equiv \angle SEB = \angle DES$$

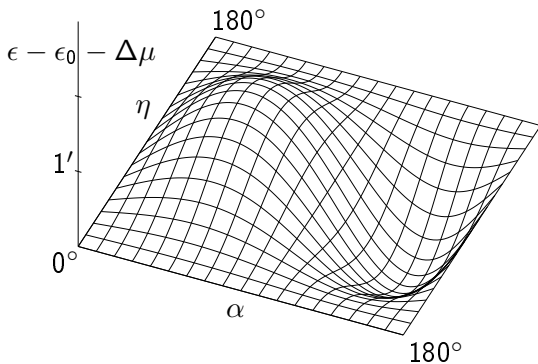
$$d(\eta) \equiv EB = \sqrt{R^2 - e^2 \sin^2 2\eta} + e \cos 2\eta$$

$$e \equiv DE, \alpha \equiv \angle ZBH$$

$$\theta \equiv \angle ZBM = \arctan \frac{e \sin 2\eta}{\sqrt{R^2 - e^2 \sin^2 2\eta} + 2e \cos 2\eta}$$

$$\epsilon(\eta, \alpha) \equiv \angle BEH = \arctan \frac{r \sin \alpha}{d(\eta) + r \cos \alpha}$$

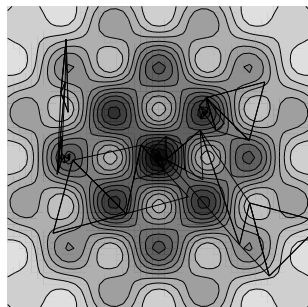
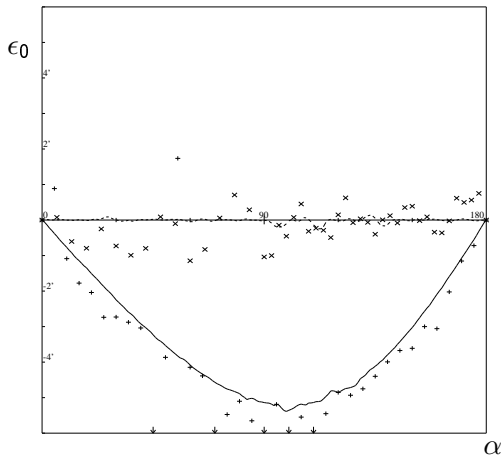
$$\simeq \epsilon_0(\alpha) + \Delta(\alpha)\mu(\eta)$$



Metoda nejmenších čtverců

$$0 = \frac{\partial}{\partial p} \sum_i w_i (y_i - f(x_i; p))^2$$

Simplexová optimalizace



- × *Almagest*
- + *Židkovy tabulky*
- - - *Toledské tabulky*
- *Alfonsinské tabulky*

quantity	parameter value	Almagest	Toledan tables	Alfonsine tables	Paulerinus' tables
$\theta(\eta)$	e/R	0.207865	0.207876	0.207881	0.207859
	0.207648	± 0.000056	± 0.000037	± 0.000029	± 0.000058
$\epsilon_0(\alpha)$	$r/(R + e)$	0.087441	0.0874928	0.0859937	0.085856
	0.0875	± 0.000028	± 0.0000017	± 0.0000010	± 0.000028
$\Delta(\alpha)$	e/R	0.2233	0.2294	0.2284	0.2392
	0.207648	± 0.0015	± 0.0013	± 0.0013	± 0.0022
	r/R	0.0986	0.0953	0.095804	0.0913
	0.105669	± 0.0007	± 0.0006	± 0.0006	± 0.0009
	cc	-0.9678	-0.9832	-0.9825	-0.91986
$\mu(\eta)$	e/R	0.18101	0.23005	0.23005	0.2504
	0.207648	± 0.0025	± 0.0025	± 0.0025	± 0.0042
	r/R	0.40741	0.0	0.0	0.59449
	0.105669	± 0.00125	± 0.00002	± 0.00005	± 0.007837
	cc	-0.7719	-0.11350	-0.11667	-0.61804

Interpolace dělením

Toruň BU 74 fol. 256v

Mūs anoz collos	Aur mōmū					Aur solier veis					Aur Saturn					Differēna			Dm dte	
	E	A	Z	S		E	A	Z	S		E	A	Z	S		A	Z	S	Z	S
1020	0	19	2	23	10	3	0	21	20	18	8	12	26	4	18	11	20	8	34	0
1000	0	19	10	3	22	3	0	39	26	20	8	12	31	24	20	11	30	2	32	30
1000	0	19	20	33	20	3	0	40	40	30	8	12	29	14	30	11	20	2	32	0

fol. 269rb:

apogea k roku 1430... nejbližší nižší 1424... přebytek 6 let...

$$[20/6] = 3 \dots 20 = 6 \cdot 3 + 2 \dots D = 11'40''8''' = 42008''' \dots$$

$$\Rightarrow \Delta \simeq D/k = 42008'''/3 = 14002''' \dots$$

$$d \equiv D/N = 35''0''' = 2100''' \dots 2100 \cdot 2/3 = 4200/3 = 1400 \dots$$

$$\Rightarrow \Delta = 14002''' - 1400''' = 12602''' = 3'30''2'''$$

alternativa násobením: $\Delta \simeq 6 * 2100''' = 12600''$

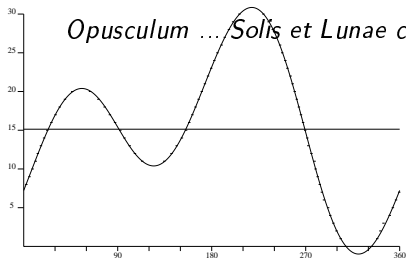
krok $N = 20$ let, difference $D = 42008'''$,

za l let přírůstek $\Delta = D * l/N = l * d$, kde $d \equiv D/N$

$$\Delta = D * l/N = D * (1/k - (N - l * k)/N/k) = D/k - d * (N - l * k)/k,$$

kde $k \equiv [N/l]$

Časová rovnice (Václav Faber z Budějovic)



Opusculum ... Solis et Lunae coniunctionum..., 1494-5

DVT 50 (2017), 192

$$\tan \alpha = \cos \varepsilon \tan l$$

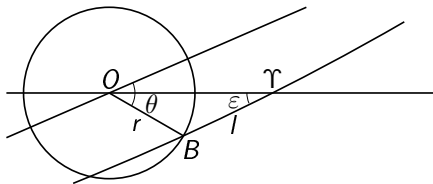
$$\tan(l - l_a) = \frac{\sin m}{\cos m + e}$$

	Faber čas	tabula vetus úhel	tabula moderna čas	Peurbach čas	Handy tables čas	Toledské tabulky úhel
Δ_0	3.908°	4.014°	4.082°	4.099°	-3.428°	4.099°
l_0	-0.1997°	-0.2429°	0.0817°	0.0290°	0.0178°	-0.1486°
e	0.03480	0.03488	0.03785	0.03779	0.04164	0.03441
ε	23.642°	23.640°	23.597°	23.570°	23.712°	23.612°
l_a	87.734°	87.628°	90.635°	90.008°	66.088°	82.890°
error	0.0363°	0.0235°	0.0252°	0.0085°	0.0132°	0.0268°

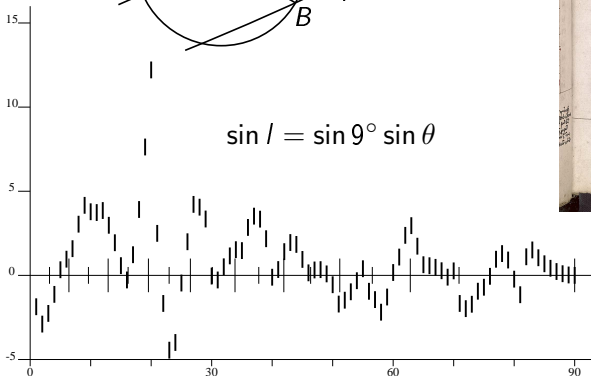
Trepidace

NK XIV F 10, XIII C 17, IG 6, XA 3, XB 3

Ratdolt 1483, Santritter 1492



$$\sin l = \sin 9^\circ \sin \theta$$



Tabula equinoctio moris arcuum recessione aequinoctio
mensis capitis Arietis sine obliquitate

Equinoctio	Equinoctio	Equinoctio	Equinoctio	Equinoctio	Equinoctio
0	0	0	0	0	0
1	0 09 2	0 09 2	0 09 2	0 09 2	0 09 2
2	0 09 24	0 09 24	0 09 24	0 09 24	0 09 24
3	0 09 48	0 09 48	0 09 48	0 09 48	0 09 48
4	0 10 12	0 10 12	0 10 12	0 10 12	0 10 12
5	0 10 36	0 10 36	0 10 36	0 10 36	0 10 36
6	0 11 00	0 11 00	0 11 00	0 11 00	0 11 00
7	0 11 24	0 11 24	0 11 24	0 11 24	0 11 24
8	0 11 48	0 11 48	0 11 48	0 11 48	0 11 48
9	0 12 12	0 12 12	0 12 12	0 12 12	0 12 12
10	0 12 36	0 12 36	0 12 36	0 12 36	0 12 36
11	0 13 00	0 13 00	0 13 00	0 13 00	0 13 00
12	0 13 24	0 13 24	0 13 24	0 13 24	0 13 24
13	0 13 48	0 13 48	0 13 48	0 13 48	0 13 48
14	0 14 12	0 14 12	0 14 12	0 14 12	0 14 12
15	0 14 36	0 14 36	0 14 36	0 14 36	0 14 36
16	0 15 00	0 15 00	0 15 00	0 15 00	0 15 00
17	0 15 24	0 15 24	0 15 24	0 15 24	0 15 24
18	0 15 48	0 15 48	0 15 48	0 15 48	0 15 48
19	0 16 12	0 16 12	0 16 12	0 16 12	0 16 12
20	0 16 36	0 16 36	0 16 36	0 16 36	0 16 36
21	0 17 00	0 17 00	0 17 00	0 17 00	0 17 00
22	0 17 24	0 17 24	0 17 24	0 17 24	0 17 24
23	0 17 48	0 17 48	0 17 48	0 17 48	0 17 48
24	0 18 12	0 18 12	0 18 12	0 18 12	0 18 12
25	0 18 36	0 18 36	0 18 36	0 18 36	0 18 36
26	0 19 00	0 19 00	0 19 00	0 19 00	0 19 00
27	0 19 24	0 19 24	0 19 24	0 19 24	0 19 24
28	0 19 48	0 19 48	0 19 48	0 19 48	0 19 48
29	0 20 12	0 20 12	0 20 12	0 20 12	0 20 12
30	0 20 36	0 20 36	0 20 36	0 20 36	0 20 36
31	0 21 00	0 21 00	0 21 00	0 21 00	0 21 00
32	0 21 24	0 21 24	0 21 24	0 21 24	0 21 24
33	0 21 48	0 21 48	0 21 48	0 21 48	0 21 48
34	0 22 12	0 22 12	0 22 12	0 22 12	0 22 12
35	0 22 36	0 22 36	0 22 36	0 22 36	0 22 36
36	0 23 00	0 23 00	0 23 00	0 23 00	0 23 00
37	0 23 24	0 23 24	0 23 24	0 23 24	0 23 24
38	0 23 48	0 23 48	0 23 48	0 23 48	0 23 48
39	0 24 12	0 24 12	0 24 12	0 24 12	0 24 12
40	0 24 36	0 24 36	0 24 36	0 24 36	0 24 36
41	0 25 00	0 25 00	0 25 00	0 25 00	0 25 00
42	0 25 24	0 25 24	0 25 24	0 25 24	0 25 24
43	0 25 48	0 25 48	0 25 48	0 25 48	0 25 48
44	0 26 12	0 26 12	0 26 12	0 26 12	0 26 12
45	0 26 36	0 26 36	0 26 36	0 26 36	0 26 36
46	0 27 00	0 27 00	0 27 00	0 27 00	0 27 00
47	0 27 24	0 27 24	0 27 24	0 27 24	0 27 24
48	0 27 48	0 27 48	0 27 48	0 27 48	0 27 48
49	0 28 12	0 28 12	0 28 12	0 28 12	0 28 12
50	0 28 36	0 28 36	0 28 36	0 28 36	0 28 36
51	0 29 00	0 29 00	0 29 00	0 29 00	0 29 00
52	0 29 24	0 29 24	0 29 24	0 29 24	0 29 24
53	0 29 48	0 29 48	0 29 48	0 29 48	0 29 48
54	0 30 12	0 30 12	0 30 12	0 30 12	0 30 12
55	0 30 36	0 30 36	0 30 36	0 30 36	0 30 36
56	0 31 00	0 31 00	0 31 00	0 31 00	0 31 00
57	0 31 24	0 31 24	0 31 24	0 31 24	0 31 24
58	0 31 48	0 31 48	0 31 48	0 31 48	0 31 48
59	0 32 12	0 32 12	0 32 12	0 32 12	0 32 12
60	0 32 36	0 32 36	0 32 36	0 32 36	0 32 36
61	0 33 00	0 33 00	0 33 00	0 33 00	0 33 00
62	0 33 24	0 33 24	0 33 24	0 33 24	0 33 24
63	0 33 48	0 33 48	0 33 48	0 33 48	0 33 48
64	0 34 12	0 34 12	0 34 12	0 34 12	0 34 12
65	0 34 36	0 34 36	0 34 36	0 34 36	0 34 36
66	0 35 00	0 35 00	0 35 00	0 35 00	0 35 00
67	0 35 24	0 35 24	0 35 24	0 35 24	0 35 24
68	0 35 48	0 35 48	0 35 48	0 35 48	0 35 48
69	0 36 12	0 36 12	0 36 12	0 36 12	0 36 12
70	0 36 36	0 36 36	0 36 36	0 36 36	0 36 36
71	0 37 00	0 37 00	0 37 00	0 37 00	0 37 00
72	0 37 24	0 37 24	0 37 24	0 37 24	0 37 24
73	0 37 48	0 37 48	0 37 48	0 37 48	0 37 48
74	0 38 12	0 38 12	0 38 12	0 38 12	0 38 12
75	0 38 36	0 38 36	0 38 36	0 38 36	0 38 36
76	0 39 00	0 39 00	0 39 00	0 39 00	0 39 00
77	0 39 24	0 39 24	0 39 24	0 39 24	0 39 24
78	0 39 48	0 39 48	0 39 48	0 39 48	0 39 48
79	0 40 12	0 40 12	0 40 12	0 40 12	0 40 12
80	0 40 36	0 40 36	0 40 36	0 40 36	0 40 36
81	0 41 00	0 41 00	0 41 00	0 41 00	0 41 00
82	0 41 24	0 41 24	0 41 24	0 41 24	0 41 24
83	0 41 48	0 41 48	0 41 48	0 41 48	0 41 48
84	0 42 12	0 42 12	0 42 12	0 42 12	0 42 12
85	0 42 36	0 42 36	0 42 36	0 42 36	0 42 36
86	0 43 00	0 43 00	0 43 00	0 43 00	0 43 00
87	0 43 24	0 43 24	0 43 24	0 43 24	0 43 24
88	0 43 48	0 43 48	0 43 48	0 43 48	0 43 48
89	0 44 12	0 44 12	0 44 12	0 44 12	0 44 12
90	0 44 36	0 44 36	0 44 36	0 44 36	0 44 36

(Pseudo)Thabit's "De motu octave spere"

Thabit ibn Qurra, ~830-901

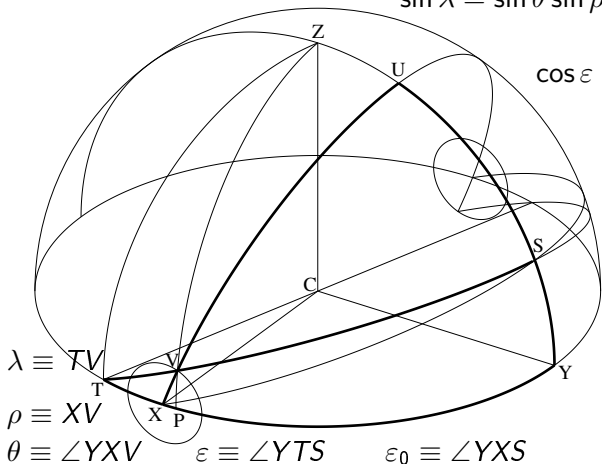
Gerard of Cremona, 1114-1187

$$\sin \lambda = \sin \theta \sin \rho \left\{ \frac{1 + \tan^2 \rho \sin^2(\theta - \varepsilon_0)}{\sin^2 \varepsilon_0 + \tan^2 \rho \sin^2(\theta - \varepsilon_0)} \right\}^{1/2}$$

$$\cos \varepsilon = \frac{\cos \varepsilon_0}{[1 + \tan^2 \rho \sin^2(\theta - \varepsilon_0)]^{1/2}}$$

R. Mercier (1976-1977)

$$\lambda \simeq \lambda_{max} \sin \theta$$



$\lambda \equiv TV$

$\rho \equiv XV$

$\theta \equiv \angle YXV$

$\varepsilon \equiv \angle YTS$

$\varepsilon_0 \equiv \angle YXS$

Menelaos $\triangle UXY \times TVS$: $\operatorname{tg} \alpha = \operatorname{tg} \rho \frac{\sin(\theta - \varepsilon_0)}{\sin \varepsilon_0}$

Cosine $\triangle TVX$: $\cos \lambda = \cos \alpha \cos \rho - \sin \alpha \sin \rho \cos \theta$

Sine $\triangle TVX$: $\sin \varepsilon = \sin \rho \frac{\sin \theta}{\sin \lambda}$

Sine $\triangle XPV, \triangle TPV$:

$$\sin \delta = \sin \theta \sin \rho = \sin \varepsilon \sin \lambda$$

Menelaos $\triangle PYZ \times XVU$:

$$\sin \alpha_V = \frac{\tan \delta}{\tan \theta}$$

Menelaos $\triangle PYZ \times TVS$:

$$\tan \varepsilon = \frac{\tan \delta}{\sin(\alpha + \alpha_V)}$$

Sine $\triangle TVZ$:

$$\sin \lambda = \sin(\alpha + \alpha_V) \frac{\cos \delta}{\cos \varepsilon}$$

